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# **Review of Research on Analysis of Functionally Graded Cylindrical Shell using Exponential Shell Theory**

**Kiran D. Shrikhande<sup>1</sup>, Govind H. Dake<sup>2</sup>, Dr. Sunil D. Shinde<sup>3</sup>, Dr. Akram S. Pathan<sup>4</sup>**

M. Tech Student, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies,  
Chhatrapati Sambhajinagar Maharashtra India<sup>1</sup>

Assistant Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies,  
Chhatrapati Sambhajinagar Maharashtra India<sup>2</sup>

Assistant Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies,  
Chhatrapati Sambhajinagar Maharashtra India<sup>3</sup>

Associate Professor, Department of Civil Engineering, Deogiri Institute of Engineering and Management Studies,  
Chhatrapati Sambhajinagar Maharashtra India<sup>4</sup>

**ABSTRACT:** Functionally Graded Materials (FGMs) are increasingly used in advanced structural applications due to their ability to smoothly transition material properties across the thickness, improving performance under thermal, mechanical, and dynamic loads. FGM cylindrical shells are critical in aerospace, marine, and nuclear industries, where accurate prediction of structural behavior is essential. This work focuses on the analysis of FGM cylindrical shells using Exponential Shell Theory, which effectively models shear deformation without requiring a shear correction factor, satisfying traction-free boundary conditions. The theory, originally proposed by Karama et al. (2008), is integrated with advanced computational methods such as the element-free kp-Ritz method, Asymptotic Numerical Method (ANM), and Nonlinear Plane Elasticity Theory (NPET) for enhanced accuracy in evaluating bending, vibration, and thermal buckling responses. The proposed framework offers a balance between computational efficiency and analytical precision, making it a powerful tool for the modeling and design of functionally graded structures under complex loading scenarios.

**KEYWORDS:** Functionally Graded Materials (FGMs), Cylindrical Shells, Exponential Shell Theory, Shear Deformation, kp-Ritz Method, Asymptotic Numerical Method (ANM), Nonlinear Plane Elasticity Theory (NPET)

## **I. INTRODUCTION**

Functionally Graded Materials (FGMs) have emerged as advanced materials with spatially varying properties that enhance the mechanical performance of structures under complex environmental conditions. Their ability to smoothly transition material properties across the thickness makes them ideal for applications involving high thermal gradients, stress concentrations, or dynamic loading, such as in aerospace, nuclear, and marine industries. FGM cylindrical shells are of particular engineering importance due to their widespread use in structural components such as pipelines, pressure vessels, and aerospace fuselages. Accurate modeling of these structures is essential to predict their behavior under various loading scenarios including bending, vibration, and buckling.

Over the years, different modeling strategies have been proposed, ranging from classical and first-order shear deformation theories to more advanced higher-order and refined theories. Karama, Afaq, and Mistou (2008) introduced a refined multilayered composite theory using an exponential shear stress distribution that satisfies free boundary conditions, eliminating the need for a shear correction factor. This approach, comparable in complexity to Touratier's sine model, represents a significant advancement in modeling the shear behavior of laminated and sandwich structures. Their model is built on a kinematic assumption and uses the virtual power principle to derive governing equations. Zhao, Lee, and Liew (2009) applied the element-free kp-Ritz method to analyze static and dynamic responses



of metal-ceramic FGM shells, highlighting the influence of boundary conditions, volume fraction, and geometric parameters. Thai and Kim (2015) provided a comprehensive review of classical and refined theories, including equivalent single-layer models and three-dimensional elasticity solutions, which laid the groundwork for improved FGM modeling strategies. Several recent developments focus on mesh-free and high-order numerical methods. Fouaidi et al. (2018) used the Asymptotic Numerical Method (ANM) within the First-Order Shear Deformation Theory (FOSDT) and a mesh-free collocation scheme to investigate buckling behavior. Bahadorani et al. (2024) extended this to nonlinear elasticity, addressing large deformations in axisymmetric FG cylindrical structures using Nonlinear Plane Elasticity Theory (NPET). Dynamic thermal effects have also received significant attention. Zhang, Chen, and Zheng (2019) analyzed dynamic thermal buckling using the Hamiltonian symplectic method, offering insights into the eigenvalue-based stability of FGM shells.

## **II. RESEARCH REVIEW**

**M Karama, K S Afaq, S Mistou (2008):** This work presents a new multilayered laminated composite structure model to predict the mechanical behavior of multilayered laminated composite structures. This new multilayered structure model describes the shear stress distribution model through the thickness respecting free boundary conditions on the top and bottom surfaces by an exponential function. This model has the same order of complexity as Touratier's model 'Sine', so a shear correction factor is not required like in the first-order shear deformation theory. This model is more precise than all other existing refined theories. This theory is based on the kinematic approach in which the shearing is represented by an exponential function. The virtual power principal is used to reduce the boundary value problem. To verify the precision of the present model, several significant problems on bending, vibration, and buckling of laminated and sandwich structures have been studied.

**X. Zhao, Y.Y.Lee, K.M.Liew (2009):** The static response and free vibration of metal and ceramic functionally graded shells are analyzed using the element-free kp-Ritz method. The material properties are assumed to vary continuously along the depth direction. The displacement field is expressed in terms of a set of mesh-free kernel particle functions according to Sander's first-order shear deformation shell theory. The effects of the volume fraction, material property, boundary condition, and length-to thickness ratio on the shell deflection, axial stress, and natural frequency are examined in detail.

**Huu-Tai Thai, Seung-Eock Kim (2015):** This review mainly focuses on the equivalent single layer theories including the classical plate theory, first order shear deformation theory, higher-order shear deformation theories, simplified theories and mixed theories since they were widely used in the modeling of functionally graded plates and shells. In addition, a thorough review of the literature related to the development of three-dimensional elasticity solutions and a unified formulation is also presented.

**Mustapha Fouaidi, Abdellah Hamdaoui, Mohammad Jamal, Bouazza Braikat(2018):** In this paper, a high order mesh-free method is investigated for buckling and post-buckling analysis of shells subjected to external loadings. This high order mesh-free method is built within the framework of the First Order Shear Deformation Theory (FOSDT) employing the Asymptotic Numerical Method (ANM) and the mesh-free collocation method. The strong form of nonlinear equations governing the equilibrium of these shells are derived by using the Collocation Scheme (CS) adopting the Total Lagrangian formulation (TL). The implementation of the boundary conditions is performed by the direct collocation method. Some benchmarks are considered to verify the ability and the accuracy of the used algorithm for investigating the buckling and post-buckling behavior of this type of structures. The obtained results are compared to those computed numerically by the same algorithm using the Finite Element Method (FEM) and to those of literature.

**D. T. Sarathchandra, S. Kanmani Subbu, N. Venkaiah (2018):** In this work, modeling and structural analysis have been carried out on a graded cylindrical shell with varying elemental composition from inner surface to the outer surface. Distribution of volume fraction of the elements is calculated using power law for cylindrical shell modeling. Static structural analysis of cylindrical shell which is exposed to internal pressure performed and results are verified with analytical solutions.

**A. H. Sofiyev (2018):** The purpose of this study is to provide an exhaustive review of the literature on the vibration and buckling of functionally graded materials (FGMs), functionally graded conical shells (FGCSs), functionally graded

layered conical shells (FGLCSs), and functionally graded sandwich-conical shells (FGSCSs). The methodological solutions for various problems encountered in pure FGCSs and FGSCSs in the design, for example, linear and non-linear (NL) vibration and stability under various loads and the influences of the different environment are presented. The examples of FGM structures included in the review cover a wide range of applications in nuclear, space and marine engineering, electronics and biomedical fields.

**Devesh Punera, Tarun Kant (2018):** has summarized the study of various aspects of design, performance and development of FG shells under the sub-sections of stress and vibration analyses. The distinction is made on the basis of type of geometry studied (i.e., circular cylinder, spherical, shallow shell, conical, etc.), theory of analysis adopted (i.e., three-dimensional (3D) elasticity approach or two-dimensional (2D) shell theories), solution procedure adopted for system of equations and volume fraction gradation considered.

**Jinghua Zhang, Shuai Chen, Wu Zheng (2019):** This study focuses on dynamic buckling of functionally graded material (FGM) cylindrical shells under thermal shock. The transient non-uniform temperature fields in the FGM shells subjected to dynamic thermal loading are determined using an analytic method. Based on the Hamiltonian principle, the dynamic thermal buckling problem of the FGM cylindrical shells is transformed into the symplectic space for solving. At the same time, the buckling thermal loads and buckling modes corresponding to generalized eigenvalues and eigen solutions of canonical equations can be calculated via the bifurcation conditions. The dynamic thermal buckling characteristics of the FGM cylindrical shells as well as the solving processes are given by the symplectic method. A complete dynamic buckling modes space is presented for the FGM cylindrical shells. The effects of the material gradient, parameters of structural geometry and thermal loadings on the dynamic buckling temperature are discussed.

**Yamna Belkhodja, Djamel Ouinas, Fatima Zohra Zaoui, Hamida Fekirini (2019):** In this study is to analyze the bending, free vibration as well as the buckling mechanical behaviors, where the results are more focused on the investigation of different parameters such as the volume fraction index, geometric ratios, frequency modes, in-plane compressive load parameters and material properties effects on the deflection, stresses, natural frequencies, and critical buckling load, which are validated in terms of accuracy and efficiency with other plate theories results found in the literature.

**Aman Garg, Mohamed-Ouejdi Belarbi, H.D. Chalak, Anupam Chakrabarti (2020):** has been carried out regarding the application of different methods and theories available in the literature for analyzing sandwich FGM structures under different loading conditions. The literature survey is carried out for sandwich FGM plates, beams, and shells. Effects of porosities, hygrothermal loadings, and structures resting on elastic foundations are also included during the review.

**Atteshamuddin S. Sayyad, Yuwaraj M. Ghugal (2021):** In this study present here using various equivalent single-layer shell theories considering the shear deformation and rotary inertia effects. The kinematics of the various shell theories are presented using a generalized shell theory in which the displacement field is independent of the selection of transverse shear strain function which results in a theoretical unification of various equivalent single-layer shell theories. The present generalized formulation yields traction free boundary conditions on the top and bottom surfaces of the shell using constitutive relations. Equations of motion are derived by using Hamilton's principle which are further solved analytically using the Navier's technique of assuming unknown variables in the double trigonometric series. Displacements, stresses, and frequencies are presented for functionally graded cylindrical, spherical, hyperbolic paraboloid, and elliptical paraboloid shells.

**T-V. Vu, H. Nguyen-Van, C.H. Nguyen, T-P. Nguyen, J.L. Curiel-Sosa (2021):** In this research present for analysis the mechanical behavior of both isotropic and sandwich FGM plates. Material properties are set to be isotropic at each point and varied across the thickness direction obeying to a power-law distribution of the volume fraction gradation with respect to FGM core or skins of the plate. Unlike high-order shear deformation plate theories based on five or more variables, the displacement field of the novel RAESDT using arctangent exponential variations in planar displacements were approximated by only four unknowns, satisfying naturally tangential stress-free conditions at the plate surfaces and leading to reduce computational efforts. In accordance with RAESDT and enhanced moving kriging interpolation (EMKI)-based meshfree method with a new quadrature correlation function is introduced for the numerical modeling. Numerical validations with different plate configurations, geometries, length to thickness ratios and boundary conditions are conducted.

**Navid Bahadorani, Mehdi Ghannad, Mohammad-Hossein Sohani, Bahman Modiri (2024):** In the present study, the governing equation of pressurized axisymmetric thick cylinders made of Functionally Graded materials (FGMs) with large deformations is derived using the Nonlinear Plane Elasticity Theory (NPET). Because of large deformations along the radial direction and hence the existence of nonlinear terms in kinematic equations, the governing equation is a nonlinear second-order equation with variable coefficients, which is solved in plane stress and plane strain states using the perturbation technique. According to the equilibrium equation, boundary conditions and different end conditions of the cylinder: open ends and closed ends; radial, axial, and circumferential stresses and radial displacement in cylindrical shells are analytically calculated.

### III. CONCLUSION

In this research review functionally graded material analysis using Exponential Shell Theory offers a robust and accurate framework for the analysis of functionally graded cylindrical shells, particularly under complex mechanical and thermal loading conditions such as bending, vibration, and thermal buckling. By incorporating an exponential representation of shear deformation, it effectively eliminates the need for shear correction factors commonly required in lower-order theories. When integrated with advanced numerical and analytical techniques such as the element-free kp-Ritz method, Hamilton's principle, and the Nonlinear Plane Elasticity Theory, this model provides enhanced predictive capability and computational efficiency. As a result, the exponential shell theory stands out as a powerful and reliable alternative to classical and first-order theories, making it highly suitable for advanced structural analysis in engineering applications involving functionally graded materials (FGMs).

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